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Estimation of Transmission Coefficient of Water Waves striking with Verticle Barrier

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Abstract: Analytical expressions for the diffracted potentials are obtained by use of the method of seperation of variables.The complex transmission coefficents are determined using eigenfunction method. The methods of analytical algebraic least-square approximation are employed to solve the corresponding over-determined system of linear algebraic equations and thereby evaluate the reflection coefficients. Results obtained using boundary element method are used to comapre absolute values of the transmission coefficients.

Keywords: Boundary value problem; least-square approximation;algebraic least-square method; transmission coefficient.

1 Introduction

The interaction of surface waves with a fixed or moving obstaclue has long standing interest in may engineering applications. Ursell[1] and others studies transmission of water waves in two dimension analytically. Among theoretical studies, Miles[3] used a scattering matrix method to calculate the reflection and transmission coefficients for the case of a step discontinuity between two finite depths. A scattering matrix obtained from the variation formulation was defined by relating the coefficients of the two propagating modes on each side of the step. Over a smoothly varying bottom topography O,Hare and Davies [5] studied propagation of waves. A similar technique was also used by Rey et al. [6]. The interaction of laminar wakes with free-surface waves generated by a moving body beneath the surface of an incompressible fluid was solved by Lu[7]using the method of integral transforms. Feng and Lu[8] analysed the problem of interaction of surface water waves with floating structures of arbitrary shapes and its solution was obtained with the aid of an eigenfunction expansion method.

Algebraic over-determined systems of equations are obtained using eigen function method and then further solve by using least- square approach.

2 Mathematical formulation

Assume that after travelling infinite distance, water waves strike with a verticle barrier over the flat bottom in the finite depth. Consider the x-axis over the free surface and z-axis vertically downward. Let a thin vertical barrier is placed at the origin in the positive z direction. As

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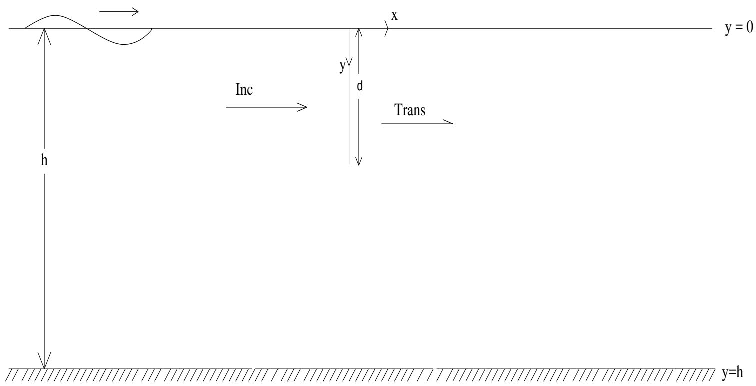

Figure 1: Sketch of scattering of surface waves by vertical barrier

the train of water waves incident upon the barrrier then some of the incident water waves are transmitted through the gap in the positive x direction (see fig. 1).

Suppose the fluid motion is irrotational and simple harmonic and the fluid is incompressible and inviscid.Here, the velocity potential $\Phi(x,z,t)$ taken as $\Phi(x,z,t) = Re\{\phi(x,z)e^{-i\sigma t}\}$. The complex velocity potential $\phi(x,z)$ satisfies the Laplace's equation:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2} = 0, \quad \text{in the fluid region} \tag{1}$$

along with the boundary conditions described as follows:

$$\frac{\partial\phi}{\partial z} + K\phi = 0 \quad \text{z} = 0, \tag{2}$$

$$\frac{\partial\phi}{\partial z} = 0 \text{ on } z = h \tag{3}$$

$$\frac{\partial\phi}{\partial x}|_{x=0^-} = \frac{\partial\phi}{\partial x}|_{x=0^+} = 0 \ \text{ on } x = 0, \ z \in L \tag{4}$$

$$\frac{\partial\phi}{\partial x}|_{x=0^-} = \frac{\partial\phi}{\partial x}|_{x=0^+} \text{ on } x = 0, \ z \in \overline{L}, \tag{5}$$

$$\phi|_{x=0^-} = \phi|_{x=0^+} \text{ on } x = 0, \ z \in \overline{L}, \tag{6}$$

$$\lim_{kx\to\infty}\left(\frac{\partial}{\partial x} \mp ik\right)\begin{pmatrix}\phi \\ \phi - \phi^{inc}\end{pmatrix} = 0., \tag{7}$$

where $\phi^{inc}(x,z)$ is the incident wave and $\overline{L}$ represents gap.

with k is the positive real root of transcendental equation $K - k\tanh kh = 0$ and $K = \dfrac{\omega^2}{g}$, ω is angular frequency of incident wave and g is gravitational acceleration. The relation (4) represents normal velocity component, i.e., the normal velocity in the x direction along the barrier being zero. The relations (5) and (6) represent continuity of velocity and pressure respectively.

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3 Method of solution

From the governing equation and boundary conditions, the velocity potentials of waves propagating in the given domain is given by

$$\phi(x,z) = A_0 e^{ikx} \frac{\cosh k(h-z)}{\cosh kh} + \sum_{n=1}^{\infty} A_n cos k_n (h-z) e^{-k_n x}, \quad x > 0, \tag{8}$$

$$\phi(x,z) = e^{ikx} \frac{\cosh k(h-z)}{\cosh kh} + B_0 e^{-ikx} \frac{\cosh k(h-z)}{\cosh kh} + \sum_{n=1}^{\infty} B_n cos k_n (h-z) e^{k_n x}, \quad x < 0 \tag{9}$$

where k_n are the roots of the equation $K + k \tan kh = 0$.

Using conditions (4) and (5), we obtain

$$\begin{aligned} ik \frac{\cosh k(h-z)}{\cosh kh} - iB_0 k \frac{\cosh k(h-z)}{\cosh kh} + \sum_{n=1}^{\infty} B_n k_n cos k_n (h-z) \\ = A_0 ik \frac{cosh\ k(h-z)}{cosh\ kh} - \sum_{n=1}^{\infty} A_n k_n cos k_n (h-z). \end{aligned} \tag{10}$$

Matching the coefficients in both side of the equation, we have

$$1 - A_0 - B_0 = 0 \qquad \text{and} \qquad A_n = -B_n. \tag{11}$$

Using the condition (4), we have

$$A_0 ik \frac{\cosh k(h-z)}{\cosh kh} - \sum_{n=1}^{\infty} A_n k_n \cos k_n (h-z) = 0, \quad z \in L$$

$$\sum_{n=0}^{\infty} A_n k_n \cos k_n (h-z) = 0, \quad z \in L \tag{12}$$

From the condition (6) across the gap, we have

$$(1 - A_0 + B_0) \frac{cosh\ k(h-z)}{cosh\ kh} + \sum_{n=1}^{\infty} (B_n - A_n) k_n cos k_n (h-z) = 0, \quad z \in \overline{L} \tag{13}$$

$$\frac{cosh\ k(h-z)}{cosh\ kh} + \sum_{n=0}^{\infty} -A_n k_n cos k_n (h-z) = 0, \quad z \in \overline{L} \tag{14}$$

On solving the equations (12) and (14) to determine the unknowns $A_n (n = 0, 1, \dots)$. Assuming z_1, z_2, z_3....... and $\overline{z}_1, \overline{z}_2, \overline{z}_3$...... discrete points respectively on barrier L and gap $\overline{L}$ to get over-determined system as given by

$$Mx = b$$

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Table 1: $|T|$ for different values of d, N, m_1 and m_2

a	(m_1, m_2, N)	T	$\|T\|$
0.2	$(50, 50, 70)$	0.9997+0.0174i	0.9998
	$(100, 100, 120)$	0.9997+0.0176i	0.9998
0.4	$(50, 50, 70)$	0.9946+0.0732i	0.9972
	$(100, 100, 120)$	0.9947+0.0724i	0.9973
0.6	$(50, 50, 70)$	0.9683+0.1748i	0.9839
	$(100, 100, 120)$	0.9687+0.1739i	0.9842
0.8	$(50, 50, 70)$	.9895-0.1607i	1.0025
	$(100, 100, 120)$	0.8678+0.3385i	0.9315

where

$$M = \begin{bmatrix} k_0h\cos k_0h(1-z_1/h) & k_1h\cos k_1h(1-z_1/h) & k_2h\cos k_2h(1-z_1/h) & \cdots \\ \cos k_0h(1-\hat{z}_1/h) & \cos k_1h(1-\hat{z}_1/h) & \cos k_2h(1-\hat{z}_1/h) & \cdots \\ k_0h\cos k_0h(1-z_2/h) & k_1h\cos k_1h(1-z_2/h) & k_2h\cos k_2h(1-z_2/h) & \cdots \\ \cos k_0h(1-\hat{z}_2/h) & \cos k_1h(1-\hat{z}_2/h) & \cos k_2h(1-\hat{z}_2/h) & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix}$$

$$x = \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ \vdots \end{bmatrix}; b = \begin{bmatrix} 0 \\ \dfrac{\cos k_0h(1-\hat{z}_1/h)}{\cos k_0h} \\ 0 \\ \dfrac{\cos k_0h(1-\hat{z}_1/h)}{\cos k_0h} \\ \vdots \end{bmatrix}$$

$$Error, E = \| Mx - b \|_2 \tag{15}$$

4 Numerical results and discussion

The values of parameters are considered in non-dimensional form using depth of water h as the fixed paramerter, such as $L = (0, \frac{d}{h})$, $\overline{L} = (\frac{d}{h}, 1)$. In $L = (0, \frac{d}{h})$, the points $\frac{z_i}{h} = \frac{z_1}{h} + (i-1)h_1, (i = 1, 2, 3....., m_1)$ with $\frac{z_1}{h} = 0, \frac{z_{m_1}}{h} = \frac{d}{h}$ and spacing $h_1 = \frac{d}{h(m_1-1)}$ are chosen. Similarly, in $\overline{L} = (\frac{d}{h}, 1)$, the points $\frac{\overline{z_i}}{h} = \frac{\overline{z_1}}{h} + (i-1)h_2, (i = 1, 2, 3...., m_2)$, with $\frac{\overline{z_1}}{h} = \frac{d}{h}, \frac{\overline{z_{m_2}}}{h} = 1$, and spacing $h_2 = (1 - \frac{d}{h})\frac{1}{m_2-1}$ are taken. In the table 1, $|T|$ are given for different barrier length and different discrete points. The table clearly shows that the values of T are complex numbers.

Here $N = 100$ is fixed throughout the numerical computation.

From the tabular data it is easy to observe that the reflection coefficient is increasing as the

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length of the barrier is increasings see table

Table 2: Error for different values for N

N	Error,E
10	0.2442
30	0.0921
50	0.0610
70	0.0472
90	0.0397
100	0.0368

5 Conclusion

The transmission coefficients are obtained using eigen function expansion method followed by algebaric least-square method. The l_2 norm used to find error in the obtained values and the obtained results are presented in the tabular form.

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